DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH350 Classical Physics **Problem Set 1** **Due: 21.08.2008**

---

1. A ray begins above a flat reflecting surface at point (x_1, y_1), travels in a straight line to the surface at $(x, 0)$, reflects and returns to a different point (x_2, y_2),
   (a) Calculate the time travel function $T(x)$.
   (b) Derive the law of reflection (angle of incidence = angle of reflection) from Fermat's Principle by solving $T'(x) = 0$.
   (c) Show that the travel time found in the previous part is an absolute minimum.

2. In deriving Snell's law from the principle of least light in class we assumed without proof that the incident and relected rays and the surface normal all lie in the same plane. Show that this assumption follows from the Principle of Least Time by allowing the rays to begin at the point $(x_1, y_1, 0)$, end at the point $(x_2, y_2, 0)$, and meet the interface (the $x-z$ plane) at the point $(x, 0, z)$. Expressing the propogation time function in terms of both x and z show that its minimization leads to the desired result.

3. The problem that started it all (Johann Bernoulli, 1696) and *the* most standard problem in the calculs of variations: Brachistochrone. ("brakhus: shortness, chronos: time (greek)). A smooth curved planar wire joins two fixed points. A bead on the wire slides without friction from rest at the upper $(x = 0, y = 0)$ to the lower endpoint $(x = x_2, y = y_2)$ under the influence of gravity. Choose down to be the positive y direction.
   (a) Show that the time T required for the bead to complete its journey is given by
   $$T = \frac{1}{\sqrt{2\,g}} \int_0^{x_2} \sqrt{\frac{1 + y'^2}{y}}\, dx.$$
   (b) Show that the curve $y(x)$ making T stationary satisfies the differential equation
   $$\frac{dy}{dx} = \sqrt{\frac{b - y}{y}}$$
   where b is a constant.
   (c) Change the variable from y to ϕ where $y = b \sin^2(\phi/2)$ and show that the above can be integrated to yield
   $$x = (b/2)(\phi - \sin(\phi)).$$
   This solution describes a *cycloid*, that is, a curve mapped out by a point on the rim of a wheel as it rolls on a flat surface.

4. A point mass is constrained to move an a massless hoop of radius a fixed in a vertical plane that is rotating about the vertical at an angular speed ω. Obtain the Lagrange equations of motion assuming that the only external forces arise from gravity. What are the constants of the motion? Show that if ω is greater than a critical value ω_0, there can be a solution in which the particle remains stationary on the hoop at a point other than at the bottom, but that if $\omega < \omega_0$ the only stationary poin for the particle is at the bottom of the hoop. What is the value of ω_0?

5. A particle, mass m, moves on the inside surface of a inverted vertical right circular cone of half-angle α. Write the Lagrangian, and the equations of motion derived from it. Find the torque of the particle about the apex, as well as the angular momentum about this point. What can you say about the constancy or otherwise of the components of the angular momentum? What are the initial conditions that lead to circular motion at a constant height?

   Write a computer program (in Matlab, Mathematica, C, Fortran, whatever) to integrate the equations of the motion and plot resulting trajectories (say the projection in the plane perpendicular to the axis of the cone) for a variety of initial conditions of your choice.