**DEPARTMENT OF PHYSICS**
**INDIAN INSTITUTE OF TECHNOLOGY, MADRAS**

**PH350 Classical Physics** **Problem Set 2** **Due: 11.09.2008**

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1. Show that a one-dimensional particle subject to the force $F = -kx^{2n+1}$, where $n$ is an integer, will oscillate with a period proportional to $A^{-n}$, where $A$ is the amplitude.

2. A particle of mass $m$ moves along the $x$ axis under the influence of the potential
$$V(x) = V_0 x^2 e^{-ax^2} \tag{1}$$
where $V_0$ and $a > 0$ are constants. Find the equilibrium points of the motion, draw a rough graph of the potential and draw the phase space portrait of the system.

3. Back to the particle in a cone of half angle $\alpha$. Now tilt the cone such that its axis makes an angle $\beta$ to the vertical direction. For a particle on the inside surface write the Lagrangian, Hamiltonian and the equations of motion.(Optional: Integrate the equations numerically as you did for the vertical cone and plot orbits. Note that the angular momentum $L_z$ is no longer conserved and the motion must admit a measure of "chaotic" orbits).

4. Show that circular orbits exist for all attractive power-law central potentials. Find the radius and total energy of the circular orbit as functions of the power of $r$ and the angular momentum.

5. Check the validity of Kepler's third law of planetary motion. Find the value of $T^2/R^3$ where $T$ is the time period and $R$ is the semimajor axis of the elliptic orbit.

6. Let $\mathbf{r}$ be the position vector, $\mathbf{J}$ the angular momentum vector and $\mathbf{p}$ the momentum vector of a mass $\mu$ in a Coulomb potential $V = -\alpha/r$, where $r$ is the magnitude of $\mathbf{r}$.
   (a) Show that the Laplace-Runge-Lenz vector $\mathbf{A} = \mathbf{p} \times \mathbf{J} - \alpha\mu\mathbf{r}/r$ is a constant of the motion.
   (b) Show that $\mathbf{A}$ lies in the plane of the orbit. From the dot product of $\mathbf{A}$ with $\mathbf{r}$ obtain the equation of the orbit and show thereby that the eccentricity is given by $e = A/\mu\alpha$ and that $\mathbf{A}$ points along the major axis of an elliptical orbit.

7. describe the qualitative features of the motion of a particle of unit mass moving in the following potentials $V(q)$. In each case sketch the phase diagram and give the equations of the separatrixes, where they exist:
   (a) $V(q) = q^{-4} - 2q^{-2}$ $(q > 0)$
   (b) $V(q) = Aq^2 e^{-q^2}$ $(A > 0)$.

8. Sketch the potential function and the contours of the Hamiltonian $H(q,p) = p^2/2 + q^4/4 - q^2/2$. Give the equation of the separatrix and provide a qualitative description of the motion. Obtain the approximate period of small oscillations in the neighbourhood of any of the stavle fixed points.

9. Find the Hamiltonians corresponding to the Lagrangians
   (a) $L = (1 - \dot{q}^2)^{1/2}$ $(|\dot{q}| < 1)$
   (b) $L(q, \dot{q}, t) = \frac{1}{2} e^{\alpha t}(\dot{q}^2 - \omega^2 q^2)$.