1. For a one-dimensional system with the Hamiltonian

\[ H = \frac{p^2}{2} - 12q^2 \]

show that there is a constant of the motion

\[ D = \frac{pq}{2} - Ht \]

2. Show that the transformation

\[ Q = \log \left( \frac{1}{q} \sin(p) \right), \quad P = q \cot(p) \]

is canonical.

3. Prove that the transformation

\[ Q_1 = q_1, \quad P_1 = p_1 - 2p_2, \quad Q_2 = p_2, \quad P_2 = -2q_1 - q_2 \]

is canonical and find a generating function.

4. Let

\[ Q_1 = q_1^2, \quad Q_2 = q_1 + q_2, \quad P_1 = P_1(q, p), \quad P_2 = P_2(q, p) \]

be a canonical transformation in two freedoms.

(a) Complete the transformation by finding the most general expressions for the functions \( P_1 \) and \( P_2 \).

(b) Find a particular choice for \( P_1 \) and \( P_2 \) that will reduce the Hamiltonian

\[ H = \left( \frac{p_1 - p_2}{2q_1} \right)^2 + p_2 + (q_1 + q_2)^2 \]

to

\[ K = P_1^2 + P_2 \]

use this to solve for \( q_1(t) \), \( q_2(t) \).

5. Determine the generator which produces infinitesimal rotation in the phase plane.
6. If \( f, g \) and \( h \) are three functions on phase space, prove that the Poisson bracket satisfies the Jacobi identity, i.e. that

\[
\{f, \{g, h\}\} + \{g, \{h, f\}\} + \{h, \{f, g\}\} = 0
\]

and the Leibnitz rule, i.e. that

\[
\{f, gh\} = g\{f, h\} + \{f, g\}h
\]