DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH350 Classical Physics **Problem Set 3**

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1. For a one-dimsional system with the Hamiltonian
$$H = \frac{p^2}{2} - 12q^2$$
show that there is a constant of the motion
$$D = \frac{pq}{2} - Ht$$

2. Show that the transformation
$$Q = \log\left(\frac{1}{q}\sin(p)\right), \quad P = q\cot(p)$$
is canonical.

3. Prove that the transformation
$$Q_1 = q_1,\ P_1 = p_1 - 2p_2,\ Q_2 = p_2,\ P_2 = -2q_1 - q_2$$
is canonical and find a generating function.

4. Let
$$Q_1 = q_1^2,\ Q_2 = q_1 + q_2,\ P_1 = P_1(q,p),\ P_2 = P_2(q,p)$$
be a canonical transformation in two freedoms.
   (a) Complete the transformation by finding the most general expressions for the functions P_1 and P_2.
   (b) Find a particular choice for P_1 and P_2 that will reduce the Hamiltonian
   $$H = \left(\frac{p_1 - p_2}{2q_1}\right)^2 + p_2 + (q_1 + q_2)^2$$
   to
   $$K = P_1^2 + P_2$$
   use this to solve for $q_1(t),\ q_2(t)$.

5. Determine the generator which produces inifinitesimal rotation in the phase plane.

6. If f, g and h are three functions on phase space, prove that the Poisson bracket satisfies the Jacobi identity, i.e. that

$$\{f, \{g, h\}\} + \{g, \{h, f\}\} + \{h, \{f, g\}\} = 0$$

and the Leibnitz rule, i.e. that

$$\{f, gh\} = g\{f, h\} + \{f, g\}h$$