DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH350 Classical Physics **Problem Set 4**

---

1. Consider the motion of a particle of mass m in the one-dimenional potential

$$V(x) = \frac{1}{2}\left(kx^2 + \alpha/x^2\right) \quad (k > 0, \alpha > 0, x > 0)$$

Show that the energy and action are related by

$$E = \sqrt{k\alpha} + 2I\sqrt{k/m}$$

and deduce that the frequency of the motion is independent of the amplitude and is $\omega = 2\sqrt{k/m}$. (We solved this differently in quiz1). Show further that r is given in terms of the angle variable θ by

$$r^2 k = E + (E^2 - k\alpha)^{1/2}\sin(\theta).$$

You may assume that

$$\int_{x_1}^{x_2} dx(-Ax^2 + 2Bx - C)^{1/2}/x = \pi(BA^{-1/2} - C^{1/2})$$

x_i being the zeros of the integrand.

2. Write the Hamilton-Jacobi equation for the 1-D simple harmonic oscillator (m=1) and solve for S. Using $S = W(q, \alpha) - \alpha t$. Leave the expression for W as an integral over q. Hence show that the solution is

$$q(t) = \frac{\sqrt{2\alpha}}{\omega}\sin[\omega(t + \beta)]$$

and identify that α is really the total energy.