DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH350 Classical Physics **Problem Set 5**

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1. Consider a system of N one-dimensional independent oscillators with the Hamiltonian
$$H = \sum_{j=1}^{N} \left(\frac{p_j^2}{2m} + K q_j^{2r} \right)$$
where $r \geq 1$ is an integer, and K is a positive constant. Find the average energy per oscillator when the system is in thermal equilibrium at a temperature T. In the case $r = 1$ find the fluctuation in the energy of a single oscillator.

2. For the molecules of air in the atmosphere, if the Hamiltonian is given to be
$$H = \sum_j \left(\frac{1}{2m}(p_{jx}^2 + p_{jy}^2 + p_{jz}^2) \; + \; mgz_j \right)$$
where you can assume that $-\infty < x, y < \infty$ and $0 \leq z < \infty$, calculate the average energy per molecule, assuming the system to be in thermal equilibrium at temperature T. Show that the fall of the density as a function of the altitude is exponential, explicitly evaluating the form and the constants. Find the mean altitude of any molecule.

3. Consider a classical system of N noninteracting diatomic molecules enclosed in a box of volume V at temperature T. The Hamiltonian for a *single* molecule is taken to be
$$H(\mathbf{p_1}, \mathbf{p_2}, \mathbf{r_1}, \mathbf{r_2}) = \frac{1}{2}(p_1^2 + p_2^2) + \frac{1}{2}K|\mathbf{r_1} - \mathbf{r_2}|^2$$
where $(\mathbf{p_1}, \mathbf{p_2}, \mathbf{r_1}, \mathbf{r_2})$ are the momenta and coordinates of the two atoms in a molecule. Find
   (a) the Helmoltz free energy of the system
   (b) the specific heat at constant volume
   (c) the mean square molecule diameter $\langle|\mathbf{r_1} - \mathbf{r_2}|^2\rangle$.

4. Prove Van Leeuwen's Theorem: The phenomenon of diamagnetism does not exist in classical physics. The following procedure maybe adopted:
   (a) If $H(\mathbf{p_1}, \ldots, \mathbf{p_N}; \mathbf{q_1}, \ldots, \mathbf{q_N})$ is the Hamiltonian of a system of charged particles in the absence of a magnetic field, then
$$H(\mathbf{p_1} - \mathbf{e}\,\mathbf{A_1}/\mathbf{c}, \ldots, \mathbf{p_N} - \mathbf{e}\,\mathbf{A_N}/\mathbf{c}; \mathbf{q_1}, \ldots, \mathbf{q_N})$$

is the Hamiltonian of the same system in the presence of an external magnetic field $\mathbf{B} = \nabla \times A$ where $\mathbf{A}_i$ is the value of $\mathbf{A}$ at position $\mathbf{q}_i$. Prove this very important fact. This is known as "minimal coupling". (e is the charge and c is the speed of light, ignore the bold type for these in the Hamitonian above!)

(b) The induced magentization of the system along the direction of B is given by

$$M = \langle -\frac{\partial H}{\partial B} \rangle = kT \frac{\partial Q_N}{\partial B}$$

where H is the Hamiltonian in the presence of B and Q_N is the partition function. (Why is the above equation true?)